Physics 214: Introduction to Quantum Mechanics
Spring 2015

Problem Set 5 Hints

5. Absence of an $E = 0$ stationary state for a particle in a box. Townsend 3.1.
You should write down the general solution inside the box $0 < x < L$, which contains two
arbitrary constants $A$ and $B$. Then impose boundary conditions. Your general solution
should not involve $k$ or $\kappa$. You should find that the $E = 0$ time-independent Schrödinger
equation inside the box is $d^2\psi(x)/dx^2 = 0$. What are two functions whose second derivatives
vanish? The general solution is the linear combination of these two solutions with coefficients
$A$ and $B$.

6. Particle in a symmetric box. Townsend 3.7.
What this problem is asking you to do is to go through steps analogous to those in class
on Friday 27 February, ignoring the fact that it is basically the same problem that we have
already solved—the same infinite square well, only shifted by $L/2$ to the left.

First write down the general form of the solution to the time-independent Schrödinger equa-
tion. You should find that the general solution is

$$\psi(x) = A\sin(kx) + B\cos(kx),$$

for the appropriate definition of $k$ in terms of energy.

Now impose the boundary conditions $\psi(x) = 0$ at $x = L/2$ and $x = -L/2$. You should
obtain two sets of solutions satisfying the boundary conditions:

Case I: $A = 0$, $B \neq 0$.
Case II: $B = 0$, $A \neq 0$.

In each case, you want to determine the allowed values of $k$ and write down the corresponding
$\psi(x)$ and energies. (The normalization constants are the same as before, $\sqrt{2/L}$.)

After having done this, verify that you can reproduce the same energies and wavefunctions
(up to a factor of $\pm 1$) simply by making the substitution $x \rightarrow x + L/2$ in the result (3.30)
obtained in class. This substitution has the effect of shifting the box from $0 < x < L$ to
$-L/2 < x < L/2$. 