Physics 214: Introduction to Quantum Mechanics
Spring 2016

Problem Set 9

Due: Fri 15 April 2016

Reading:

Mon 4/11: Sec. 6.2.

Mon 4/13: Class notes on orbitals and spherical harmonics.

Wed 4/15: Secs. 6.3.

Problems (continued on page 2):


2. Transmission through a rectangular barrier: classically allowed case $E > V_0$.
   (a) Townsend Problem 4.27. Please do this problem using the trick described in class and in the Rec 11 handout: Write down the form of the wavefunction $\psi(x)$ in all three regions, and note that it can be obtained from that in $E < V_0$ case by the substitution $\kappa = -ik_0$. Therefore, the result for $A/C$ can be obtained from that in the previous problem by the same substitution.

   (b) Can you think of an intuitive physical explanation for why the transmission coefficient goes through successive relative maxima and minima as $k$ (or equivalently, the de Broglie wavelength $\lambda = 2\pi/k$) is varied? Hint: see part (c) for inspiration.

   (c) Let $R'$ and $T' = 1 - R'$ denote the reflection and transmission coefficients for the step potential as given by Eq. (4.121) and (4.120). Please express the transmission coefficient $T$ found in part (a) in terms of $R'$. Compare your answer to the result proven for photons on Problem 5 of Problem Set 3:

$$P = \frac{1}{1 + \frac{4|r|^2}{(1 - |r|^2)^2} \sin^2(k'd)},$$

(1)

where $r$ is the photon reflection amplitude at each interface of a thin glass plate of thickness $d$. Here, $P$ is the probability of transmission of a photon through the glass plate. (Note that the
reflection coefficient for the photon at each interface is the square of the reflection amplitude, $|r|^2$ at that interface.)

3. Transmission through a rectangular potential well. Townsend Problem 4.28. In Part (c), there are many possible values of the depth of the well $V_0$, one for each $n$. Please find a general formula for $V_0$ as a function of $n$. Express your formula in eV using the numerical data given. Then quote the minimum possible value of $V_0$ (at $n = 1$) in eV.

4. Practice with commutators. Please prove the following identities, give two operators $\hat{A}$ and $\hat{B}$:

(a) $[\hat{A}, \hat{B}] = -[\hat{B}, \hat{A}]$.

(b) $[\hat{A}\hat{B}, \hat{C}] = [\hat{A}, \hat{C}]\hat{B} + \hat{A}[\hat{B}, \hat{C}]$. This is the “product rule.”

(c) Using the results of (a) and (b) (and not starting from first principles), show that

$$[\hat{A}\hat{B}, \hat{C}\hat{D}] = [\hat{A}, \hat{C}]\hat{D}\hat{B} + \hat{C}[\hat{A}, \hat{D}]\hat{B} + \hat{A}[\hat{B}, \hat{C}]\hat{D} + \hat{A}\hat{C}[\hat{B}, \hat{D}].$$

(d) In 3D, the canonical commutation relations are $[x, \hat{p}_x] = [y, \hat{p}_y] = [z, \hat{p}_z] = i\hbar$. All other commutators of any pair of $x, y, z, \hat{p}_x, \hat{p}_y, \hat{p}_z$ vanish. Using identity (c) and the canonical commutation relations, show that

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z,$$

where $\hat{L} = \mathbf{r} \times \hat{p}$ is the angular momentum operator, given in components in Townsend Eq. (6.21).

5. Feedback. By Monday 18 April please send me an email message to provide feedback on the class and on your reading. (My email address is mbschulz at brynmawr.edu). For example: Which parts were easier or harder to understand? Do you have any questions that you would like to clarify or areas where you would like more practice in recitation section? Was there something that you found particularly interesting or uninteresting? Was the problem set of reasonable length and difficulty. If you have any thoughts on how to improve the textbook for future students using future editions, please let me know and I will pass that information on to the author, John Townsend. The purpose of the feedback is to help you to reflect on your learning process and to provide me with brief but valuable information that will help to make this class the best possible experience for everyone.