1. Which of the above wavefunctions is a possible bound state of definite energy for a finite square well?

- A. only 1
- B. only 2
- C. 1 and 2
- D. 1 and 3
- E. all of the above

2. Choose all of the following statements that are correct about an electron in a bound state of definite energy in a finite square well.

1) Its wavefunction is nonzero only inside the well.  
2) Its energy levels are discrete.  
3) Its wavefunctions are normalizable.

- A. only 1
- B. only 2
- C. only 3
- D. 1 and 2
- E. 2 and 3
- F. All of the above.
3. Solutions to the Schrödinger equation for constant potential. Consider the time-independent Schrödinger equation in a region of $x$ for which $V(x) = V_0 = \text{constant}$. Let us define

$$k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \quad \text{when } E > V_0 \quad (\text{classically allowed}),$$

and

$$\kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} \quad \text{when } E < V_0 \quad (\text{classically forbidden}).$$

(a) What is the time-independent Schrödinger equation in this interval, for $E > V_0$, $E = V_0$, and $E < V_0$. Express your answer in terms of $k$ or $\kappa$ for $E \neq V_0$.

(b) What is the general solution in each case?

**Solution.**

(a) The time-independent Schrödinger equation is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x).$$

In a region of space where $V(x) = V_0 = \text{constant}$, this becomes

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V_0\psi(x) = E\psi(x),$$

or equivalently,

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V_0)}{\hbar^2}\psi(x) = 0.$$

Therefore,

$$\begin{cases}
    \frac{d^2\psi}{dx^2} + k^2\psi(x) = 0, & k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \quad E > V_0, \\
    \frac{d^2\psi}{dx^2} = 0 & E = V_0, \\
    \frac{d^2\psi}{dx^2} - \kappa^2\psi(x) = 0, & \kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} \quad E < V_0.
\end{cases}$$

(b) The general solution in each of the three cases is

$$\begin{cases}
    \psi(x) = A\sin kx + B\cos kx & \Rightarrow \quad \psi(x) = Ce^{ikx} + De^{-ikx} \quad E > V_0, \\
    \psi(x) = Ax + B & E = V_0, \\
    \psi(x) = A\sinh \kappa x + B\cosh \kappa x & \Rightarrow \quad \psi(x) = Ce^{\kappa x} + De^{-\kappa x} \quad E < V_0.
\end{cases}$$