Electromagnetism, Relativity & Particles — Midterm Exam

1. Electrostatic Force

A uniformly charged solid sphere (radius $R$, charge density $\rho$) is composed from two hemispheres. Find the force $F$ acting between the hemispheres. Express your result also in terms of the total charge $Q$ of each hemisphere.

2. Cathode Ray Tube

TV sets usually use cathode ray tubes (CRT) to display the image. These are evacuated glass tubes with a built-in electron source. In a first step, the electrons are accelerated using high voltage $V$; then, the electron beam is deflected in a magnetic field $B$. The electron beam is quickly moving back and forth across the fluorescent screen and in this way “writes” the image, usually thirty times per second.

(a) Typically, the accelerating voltage is $V = 25 \text{ kV}$. Calculate the velocity $v$ of the electrons. (Ignore relativistic effects.)

(b) In order to reach the left margin of the screen, the electron beam has to be deflected sideways by a distance $\Delta x = 25 \text{ cm}$. Assuming that the vertical distance between the electron source and screen $\Delta z$ is also $\Delta z = 25 \text{ cm}$ (see figure), and that the magnetic field $B$ is uniform, find its strength $B$.

3. Fields and Potentials of Charged, Moving Sheet

Consider a uniformly charged, thin sheet of infinite extension (charge density $\sigma$) that is located in the $x - y$ plane, and moves with uniform speed $v$ in the $x$ direction. Find (in vectorial form) the electric and magnetic fields $E(z)$ and $B(z)$ produced by the charge distribution using Gauss’ and Ampère’s laws, and determine a corresponding electric potential $\Phi(z)$ and vector potential $A(z)$.
4. Magnetic Field of a Toroid

A toroid is a doughnut-shaped object with rotational symmetry. In this exercise, we examine the magnetic field generated by a thin wire that is tightly wound around a hollow cylindrical surface with inner radius $r$ and outer radius $R$ (a toroidal coil) — see figure.

(a) Assuming that the wire carries a current $I$ and loops $N$ times around the toroid, covering it uniformly, determine the current surface density $j(\rho)$ as a function of the distance from the symmetry axis $\rho$.

(b) Use Ampère’s law to find the magnetic field $B(\rho)$ both inside and outside the coil. Show that:

$$B(\rho) = \begin{cases} \frac{\mu_0 N I}{2\pi \rho} & \text{ (inside coil)}, \\ 0 & \text{ (outside coil)}, \end{cases}$$

where $\rho$ again denotes the lateral distance from the symmetry axis.

(c) The energy density of a magnetic field is given by $u(r) = B(r)^2/(2\mu_0)$. Set up a formula for the amount of magnetic energy $E$ stored in the toroidal coil. Evaluate $E$ for $r = 5 \text{ cm}$, $R = 10 \text{ cm}$, $z = 5 \text{ cm}$ (height of coil), $I = 1 \text{ A}$, and $N = 1000$. What is the maximum magnetic field $B$ produced by the coil under these circumstances?