Electromagnetism, Relativity & Particles — Homework Problems

1. **Resolution of Particle Wave** (10 P.)

   The resolution of any microscope is limited by the wavelength \( \lambda \) of the radiation used. Find the wavelength in a

   - beam of 1 keV synchrotron radiation (photons),
   - a 25 keV electron microscope,
   - the 20 GeV electron beam used in the Stanford linear accelerator (SLAC).

2. **Center of Mass Energy** (20 P.)

   In particle physics, one is usually studying the collision between particles in the center-of-mass system, a frame of reference which is characterized by the fact that the total momentum \( p_{cm} \) of all particles vanishes in it. The combined energy of all particles in this frame is called the center-of-mass energy \( E_{cm} \). In the early days of particle physics, beams of accelerated particles were sent against a fixed target, but today virtually all experiments take place in colliders, where two beams of particles intersect each other at high speed (despite the fact that it is not easy to hit a moving target head-on!). An analysis of \( E_{cm} \) in both setups explains why.

   (a) Calculate \( E_{cm} \) for:

   - 100 MeV betatron electrons, hitting an electron that is at rest,
   - 100 MeV betatron electrons, hitting a proton that is at rest,
   - 20 GeV SLAC electrons, hitting a proton that is at rest,
   - 20 GeV electrons in a SLAC beam, colliding with a 20 GeV positron beam that travels in the opposite direction.

   (b) The first antiprotons were observed in an experiment where a proton beam was sent into a hydrogen target at rest: A moving proton hits a proton at rest, and occasionally a proton-antiproton pair is created in the collision. Show that the threshold for this reaction (i.e., the minimum kinetic energy of the incoming proton) in the lab system is six times the rest energy of the proton, \( T = 6m_p c^2 \). (Hint: What is the threshold energy in the center-of-mass system? Use the invariant energy \( s = E^2 - p^2 c^2 \) to switch between the various frames of reference — cm frame, lab frame, and the rest frame of the incoming proton.)

3. **Compton Effect** (20 P.)

   The Compton effect is one of three mechanisms that cause high-energy photons to lose energy when interacting with matter. (The other two are the photoeffect, where an atom absorbs the photon, and an electron is set free, and pair production, where the energy of a photon is used to create a charged particle-antiparticle pair.) In the Compton effect, the energy loss takes place by elastic scattering of the photon at an electron that is initially at rest. After the collision, the electron is moving, and the photon is deflected and lost some of its energy to the electron. Consider now the particular case that the incoming photon (energy \( E_\gamma \)) is scattered under a right angle and travels on with energy \( E'_\gamma \), while the electron moves away under an angle \( \theta \) with respect to the original photon direction:

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Use conservation of energy and momentum to calculate the energy of the scattered photon, and the scattering angle for the electron, in terms of the original photon energy $E_\gamma$ and the electron rest energy $E_0 = m_e c^2$, and show that:

$$\frac{1}{E'_\gamma} = \frac{1}{E_\gamma} + \frac{1}{E_0}, \quad \tan \theta = \frac{E_0}{E_0 + E_\gamma}.$$ 

4. **Neutral Pion Decay**

The neutral pion $\pi^0$ is a short-lived particle that commonly occurs in collisions between protons, and that decays quickly into a pair of photons:

$$\pi^0 \rightarrow 2\gamma, \quad \tau \sim 10^{-18} \text{ s}.$$ 

Assume that in an experiment the photons are detected under equal angles of 60 degrees with respect to the direction of motion of the pion (see figure):

(a) Describe the decay in the rest frame of the pion. (5 P.)

(b) Find the original speed of the pion in the laboratory frame. (10 P.)