Physics 325: Elementary Particle Physics
Spring 2010

Problem Set 2

Due: Thu 4 Feb 2010

Reading: Please read Chapter 4 in Griffiths.

Problems:

3.16. Threshold energy for the process \( A + B \to C_1 + C_2 + \ldots + C_n \).

3.18. Minimum kinetic energy to make an \( \Omega^- \). This is an application of the previous problem.

3.19. Kinematics of a two body decay \( A \to B + C \).

3.25(a) and 3.26. Mandelstam variables. This is mainly algebra, but we will use the results later in the semester.

Car in barn paradox. Marty McFly has reached cruising velocity \( v = (3/5)c \) in Dr. Emmet Brown’s modified DeLorean automobile. He approaches a barn where Doc is preparing to make some measurements. Both the DeLorean and the barn have proper length \( L \). Due to the secrecy of his work, Doc will only open the door of one side of the barn at a time, relative to his frame—the rest frame of the barn. On the other hand, due to his inventiveness, Doc is able to open and close the doors in arbitrarily rapid succession, so that in his frame, at the instant the tail end of the DeLorean enters the barn, the barn door immediately closes behind it and the other one opens. In addition to the danger inherent in traveling well over the legal speed limit (which we will neglect), there is also Plutonium on board the automobile. Therefore, any collision spells disaster!

(a) What are the lengths of the DeLorian and the barn in Marty’s frame and in Doc’s frame?

(b) Does Marty make it through alive? Given your answer to part (a), explain why naively there is a paradox—why Doc and Marty’s points of view naively lead to different conclusions. Then resolve the paradox. Show that Marty and Doc in fact agree. In your explanation, please refer to a spactime diagram as well as explicit computations.
Derivation of Lorentz transformations. In this problem, you will derive the Lorentz transformations from the rules for time dilation and length contraction. Let \( S' \) denote a reference frame moving at velocity \( v \) with respect to another frame \( S \). The origins of \( S' \) and \( S \) are chosen to coincide at \((ct', x') = (ct, x) = (0, 0)\). Let \( L \) denote the position \( x' = 0 \), and let \( R \) denote a position on the \( x' \)-axis with coordinate \( x' > 0 \). For fixed \( x' \), the positions \( L \) and \( R \) can be thought of as defining, respectively, the left and right ends of a rod of length \( x' \), where \( S' \) is the rest frame of the rod.

(a) In frame \( S \), express the length of the rod in terms of \( t, x, v \), where \( x \) is the instantaneous spatial coordinate of the right end \( R \) of the rod at time \( t \). Then, using the formula for length contraction, express the same length in terms of \( x' \). Equate the two results to derive the Lorentz transformation

\[
x' = \gamma(x - vt), \quad \text{where} \quad \gamma = 1/\sqrt{1-v^2/c^2}. \tag{1}
\]

By symmetry, deduce the inverse Lorentz transformation that expresses \( x \) in terms of \( t', x' \).

(b) At time \( t'_1 \) in frame \( S' \), a light pulse is sent out from the left end \( L \) of the rod. The pulse reaches the right end \( R \) at time \( t' \). Now carry out the following three steps: (i) Express \( t'_1 \) in terms of \( t' \) and \( x' \). (ii) Let \( \Delta t \) denote the time of transit of the light pulse in frame \( S \). Show that \( \Delta t = \gamma^{-1}x'/c - v \). Since \( t_1 = t - \Delta t \), this gives an expression for \( t_1 \). (iii) Finally, using the formula for time dilation, write an equation relating \( t_1 \) to \( t'_1 \), then eliminate \( t_1 \) and \( t'_1 \) from this equation using the results of (i) and (ii). Show that solving for \( t \) gives the inverse Lorentz transformation

\[
t = \gamma(t' + vx'/c^2). \tag{2}
\]

By symmetry, deduce the Lorentz transformation that expresses \( t' \) in terms of \( t \) and \( x \).