Due: Thu 26 Jan 2012, in class.

Reading: For this week, please read Chapters 1 and 4 in Hartle. For next week, please read Chapter 5.

Problems:

1. Car in the barn paradox. Marty McFly has reached cruising velocity $v = (3/5)c$ in Dr. Emmet Brown’s modified DeLorean automobile. He approaches a barn where Doc is preparing to make some measurements. Both the DeLorean and the barn have proper length $L$. Due to the secrecy of his work, Doc will only open the door of one side of the barn at a time, relative to his frame—the rest frame of the barn. On the other hand, due to his inventiveness, Doc is able to open and close the doors in arbitrarily rapid succession, so that in his frame, at the instant the tail end of the DeLorean enters the barn, the barn door immediately closes behind it and the other one opens. In addition to the danger inherent in traveling well over the legal speed limit (which we will neglect), there is also Plutonium on board the automobile. Therefore, any collision spells disaster!

   (a) What are the lengths of the DeLorian and the barn in Marty’s frame and in Doc’s frame?

   (b) Does Marty make it through alive? Given your answer to part (a), explain why naively there is a paradox—why Doc and Marty’s points of view naively lead to different conclusions. Then resolve the paradox. Show that Marty and Doc in fact agree. In your explanation, please refer to a spactime diagram as well as explicit computations.

2. Gedankenexperiment. In the distant future, a train passes through Central Station at near light speed. Let $S$ denote the frame of the station platform and $S'$ the frame moving with the train at velocity $v$. There are three equally spaced clocks on the platform and three equally spaced clocks on the train—located at the front, center and rear in either case. The clocks are such that when the center clock on the train is just passing the center clock on the platform, they both read 12:00 noon. At that instant, a lightbulb flashes in the center of the train, emitting a pulse that propagates to the left and right. The pulse is observed to
reach the two ends of the station platform at the same time in frame $S$. It is also observed to reach the two ends of the train at the same time in frame $S'$ (why?)

(a) Based on this information and without using Lorentz transformations, draw a diagram indicating the train, platform, and times on all clocks from the point of view of frame $S$ at 12:00 noon. (For the times, please indicate 12:00, <12:00, or >12:00.) Provide a brief explanation of your diagram.

(b) Now check that your answer to part (a) agrees with the result obtained using Lorentz transformations: what is $t = 12:00 = \text{const}$, expressed in terms of $t', x'$?


4. Derivation of Lorentz transformations. In this problem, you will derive the Lorentz transformations from the rules for time dilation and length contraction. Let $S'$ denote a reference frame moving at velocity $v$ with respect to another frame $S$. The origins of $S'$ and $S$ are chosen to coincide at $(ct', x') = (ct, x) = (0, 0)$. Let $L$ denote the position $x' = 0$, and let $R$ denote a position on the $x'$-axis with coordinate $x' > 0$. For fixed $x'$, the positions $L$ and $R$ can be thought of as defining, respectively, the left and right ends of a rod of length $x'$, where $S'$ is the rest frame of the rod.

(a) In frame $S$, express the length of the rod in terms of $t, x, v$, where $x$ is the instantaneous spatial coordinate of the right end $R$ of the rod at time $t$. Then, using the formula for length contraction, express the same length in terms of $x'$. Equate the two results to derive the Lorentz transformation

$$x' = \gamma(x - vt), \quad \text{where} \quad \gamma = 1/\sqrt{1 - v^2/c^2}. \quad (1)$$

By symmetry, deduce the inverse Lorentz transformation that expresses $x$ in terms of $t', x'$.

(b) At time $t_1'$ in frame $S'$, a light pulse is sent out from the left end $L$ of the rod. The pulse reaches the right end $R$ at time $t'$. Now carry out the following three steps: (i) Express $t_1'$ in terms of $t'$ and $x'$. (ii) Let $\Delta t$ denote the time of transit of the light pulse in frame $S$. Show that $\Delta t = \gamma^{-1}x'/(c - v)$. Since $t_1 = t - \Delta t$, this gives an expression for $t_1$. (iii) Finally, using the formula for time dilation, write an equation relating $t_1$ to $t_1'$, then eliminate $t_1$ and $t_1'$ from this equation using the results of (i) and (ii). Show that solving for $t$ gives the inverse Lorentz transformation

$$t = \gamma(t' + vx'/c^2). \quad (2)$$

By symmetry, deduce the Lorentz transformation that expresses $t'$ in terms of $t$ and $x$. 