Physics 325: General Relativity  
Spring 2015  
Problem Set 5

Due: Fri 6 Mar 2015

Reading: The reading assignment is the same as last week. Please continue to read Chapters 8 and 9.

Problems (continued on other side):

1. Toy model of a black hole. Hartle Problem 7.5.

2. Metric for a 3-sphere. Hartle Problem 7.19. When we discussed the maximally curved 3D spaces in class, we wrote the metric as

\[ dS^2 = \frac{dr^2}{1 - kr^2/a^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \]  

Here, the quantity \( k \) gives the sign of the curvature. The values \( k = +1, 0, -1 \) correspond to the three-sphere, flat space and hyperbolic space, respectively. Another useful way to write the three metrics is

\[
\begin{align*}
 dS^2 &= d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2) & \text{3-sphere,} \\
 dS^2 &= d\chi^2 + \chi^2(d\theta^2 + \sin^2 \theta d\phi^2) & \text{flat space,} \\
 dS^2 &= d\chi^2 + \sinh^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2) & \text{hyperbolic space.}
\end{align*}
\]  

In this problem, you will derive the first of these expressions by embedding the 3-sphere in flat 4D Euclidean space.

3. Minkowski metric in Penrose diagram coordinates. Hartle Problem 7.6. You should find that

\[ ds^2 = \Omega^2(t', r')(-dt'^2 + dS^2_{\text{3-sphere}}(r', \theta, \phi)), \]  

where \( \Omega^2(t', r') \) is some function to be determined and \( dS^2_{\text{3-sphere}}(r', \theta, \phi) \) is as given by Eq. (2) above, with \( \chi \) replaced by \( r' \).
4. 3D Euclidean embedding of a 2D slice of the Schwarzschild black hole.
Hartle Problem 7.20. This is the cover illustration of the textbook. You can find the full
Schwarzschild metric on the inside front cover. Feel free to sketch the surface or plot it with
Mathematica, whichever you prefer.