the shadow patch; it quickly broadens, leaving finally two shadow patches, separating from each other with large velocity (arbitrarily large at $T_{\text{ad}}$).

Figure 4 shows the intensity distribution in the $XZ$ plane, at two different times $T$ and $T'$, for $\beta = 10^{-2}$. Note that the line $A_0A_1$ is perpendicular to the direction $AS$. The interesting behavior appears on a plane parallel to $OX$, in the vicinity of $A_0$ at $T$, or $A_1'$ at $T'$.

III. DISCUSSION

When $\beta_M < 1$, the finite transit time effects do not change the qualitative behavior of the eclipse, except for the nonlinearity in the curve $X(T)$. But for $\beta_M > 1$, a second shadow patch appears, the behavior of which is dominated by light propagation delays. The fact that the shadow limits move with arbitrarily large (superluminal) velocity near $X_0$, involve no anticausal behavior. These superluminal "objects," that could be visualized by putting at distance $Z$ a diffusing or fluorescent screen, are of the same nature as those produced for instance by a rotating (angular velocity $\omega$) source on a cylindrical screen at distance larger than the so-called light cylinder $R = c/\omega$. Though the superluminoius velocity is real (and is not an apparent velocity seen by an observer), it cannot be used to transmit information. One sees also that the time duration of the occultation, at a given location $X_0Z$ has the usual behavior.

These results arise in a Gedanken experiment. Could they be observed in practice? Similar behavior, obtained with an imaging system with high magnification ($M$) has been discussed in the context of high-speed photography. Can these effects be observed in actual astronomical occultations? This would require both satisfying the peculiar geometrical conditions to observe near $A_0$ or $B_0$, and measure the spatial and temporal intensity distribution in a portion of plane with the correct orientation. Furthermore, one would actually observe diffraction fringes instead of the geometrical optics shadow limit; since the latter satisfies $dX/dZ \to \infty$ near $A_0$ or $B_0$, the spatial interfringe in a plane perpendicular to $OZ$ is expected to be very large in the vicinity of these points. In any case, this is a simple instructive example where an apparently trivial situation ($\nu/c < 1$, though $M\nu/c > 1$), yields surprising results.

1See, for instance, V. L. Ginzburg, *Theoretical Physics and Astrophysics* (Pergamon, New York, 1979) and references therein.

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Undergraduate experiment: Determination of the band gap in germanium and silicon

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A method for determining the band gap in germanium and silicon at 0 K based upon the temperature dependence of the electrical conductivity of a $p$-$n$ junction is described. Results are given for the band gaps that are in good agreement with the accepted values.

I. INTRODUCTION

A central feature in the explanation of the electrical characteristics of semiconductor devices is that of a forbidden range of energies between the valence and conduction bands. This paper describes a procedure for determining the band gap in germanium and silicon that may be performed as an undergraduate experiment using readily available $p$-$n$ junction devices.

The approach adopted is similar to that described by Canivez. Modifications have been made to the analysis of the data which has resulted in more accurate values for the band gaps being obtained.

II. THEORY

The current-voltage characteristics of a $p$-$n$ junction may be represented by the junction equation found in many solid-state text books:

$$I = I_0[\exp(eV/kT) - 1],$$

where $V$ is the potential difference across the junction, $e$ is the charge on an electron, $k$ is Boltzmann's constant, $T$ is the temperature of the junction in Kelvins, and $I_0$ is the current that flows through the junction when it is reverse biased.

For an abrupt junction (where the doping profile across
the junction shows a sharp discontinuity) \( I_0 \) can be written as

\[
I_0 = A T^{(3 + \gamma/2)} \exp\left[ -E_g(T)/kT \right],
\]

where \( A \) is a constant. \( \gamma \) is a constant that depends upon the temperature dependence of the mobility, lifetime, and diffusion coefficient of minority carriers. \( E_g(T) \) is the minimum energy gap between the valence and conduction bands.

The energy gap of a semiconductor is temperature dependent. The temperature dependence may be written, to a first approximation, as

\[
E_g(T) = E_g(0) - aT,
\]

where \( E_g(T) \) is the energy gap in electron volts, \( E_g(0) \) is the energy gap at 0 K, and \( a \) is a constant.

Combining Eqs. (1), (2), and (3), taking natural logs and rearranging gives

\[
eV = E_g(0) + [k \ln(I/A) - a]T - k(3 + \gamma/2)lnT,
\]

where it is assumed that \( eV > kT \).

Throughout the rest of the experiment, the current \( I \) was held constant.

A value for \( \gamma \) may be obtained by referring to data books for the temperature dependence of mobility and diffusion coefficient of the materials studied. However such data is normally only valid for pure specimens. The temperature dependencies of the quantities under consideration are sensitive to impurity concentration, therefore any value arrived at for \( \gamma \) is likely to be approximate at best. In the experiment described here a value for \( \gamma \) was found by performing a least-squares fit to the experimental data.

III. LEAST-SQUARES FITTING TO DATA

Equation (4) can be rewritten as

\[
eV = E_g(0) + \alpha T - \beta T \ln T,
\]

where \( \alpha \) incorporates the two unknowns \( A \) and \( a \). Since Eq. (5) is linear in \( E_g(0), \alpha, \) and \( \beta \), it is possible to determine their optimal values by performing a linear least-squares fit to the experimental data. The least-squares criterion requires the minimization of \( \sum_{i=1}^{n} [V_i - V_c(i)]^2 \), where \( V_i \) and \( V_c(i) \) are the \( i \)th experimental and calculated values of voltage, respectively, and \( n \) is the number of data points. Taking derivatives of this function with respect to \( E_g(0), \alpha, \) and \( \beta, \) and equating to zero leads to the three normal equations,

\[
nE_g(0) + \alpha \sum T - \beta \sum T \ln T - \sum V_0 = 0,
\]

\[
E_g(0) \sum T + \alpha \sum T^2 - \beta \sum T^2 \ln T - \sum V_0 T = 0,
\]

\[
E_g(0) \sum T \ln T + \alpha \sum T^2 \ln T - \beta \sum T^2(\ln T)^2 - \sum V_0 T \ln T = 0.
\]

Equations (6)–(8) may be written in matrix form as

\[
\begin{bmatrix}
\sum T & -\sum T \ln T \\
\sum T & -\sum T^2 \ln T \\
\sum T \ln T & -\sum T^2(\ln T)^2
\end{bmatrix}
\begin{bmatrix}
E_g(0) \\
\alpha \\
\beta
\end{bmatrix}

= \begin{bmatrix}
\sum V_0 \\
\sum V_0 T \\
\sum V_0 T \ln T
\end{bmatrix}
\]

or

\[
Mx = v.
\]

The vector of parameter values required for minimization is then found from

\[
x = M^{-1}v.
\]

Generation of the elements of the matrix \( M \) and its inversion are tedious to calculate by hand but are easily executed by a microcomputer.

IV. EXPERIMENTAL METHOD

The experiments were performed on the emitter junctions of a silicon transistor (BC109) and a germanium transistor (AC127) (equivalents 2N930 and 2N2430, respectively).

A constant current source was connected to the device under investigation and the potential difference between the base and emitter terminals was recorded as a function of temperature. Figure 1 shows the electrical circuit that was constructed to supply the constant current. The current provided by this arrangement was in the range of 100 \( \mu A \) to 1 mA. The stability of the current was quite adequate at better than \( \pm 0.5\% \).

A spot-welded chromel-alumel thermocouple was attached to the device and both were clamped into a copper block. The thermocouple was attached to a zero point compensated thermocouple amplifier (ADS95AD) which provided an output convenient for interfacing to a microcomputer. Zinc-oxide-filled silicone was used around the device and thermocouple to ensure good thermal contact.

![Fig. 1. Circuit diagram of constant current source.](image-url)
with the copper block. The copper block was then placed within an open-neck dewar. Liquid nitrogen was introduced into the dewar and measurements of temperature and voltage were made between approximately 120 K and room temperature.

A microcomputer connected to a calibrated 10-bit analogue to digital converter was used to log the data. Additionally the microcomputer was programmed to perform graph plotting and least-squares analysis of the data.

V. RESULTS

Figure 2 shows a typical set of recorded data of $eV$ vs $T$ for the germanium and silicon devices, where

$$eV = E_g(0) + \alpha T - \beta T \ln T.$$  \hspace{1cm} (12)

The line through the points represents the least-squares fit to the data.

Table I contains the values of $E_g(0)$ and $\gamma$ based upon the linear least-squares fit. Additionally Table I contains the accepted values of the energy gaps for germanium and silicon.\(^6\)

VI. CONCLUSION

The values obtained for the energy gaps of silicon and germanium are in good agreement with the accepted values. The experiment can be performed with equipment readily available in an undergraduate physics laboratory. The devices used are inexpensive and easily obtainable.

The approach described here could be extended to other $p$-$n$ junction devices such as those formed fromGaAs and GaP which are readily available as light-emitting diodes.

Finally, in addition to determining the energy gap of a semiconductor, the experiment provides the student with a valuable insight into the theory of the $p$-$n$ junction.

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