Introduction
In an earlier lab you studied the emission spectra of different elements excited by a gas discharge: hydrogen, deuterium, helium, sodium, mercury. There are two common ways of separating light by its wavelength so that such spectra can be measured. One uses a prism, in which the dependence of the index of refraction of the glass on wavelength means that there are different angles at which the light rays are bent. After passing through two sides of the prism, the incident light is separated into its colors, each of which travels in a different direction. The second way uses a grating, light scattered from each of the rulings on the grating will interfere in specific directions depending on the angle of incidence of the light and the spacing of the rulings on the grating. In this lab, you will use a commercial monochrometer, that uses such a grating, to acquire dispersed emission spectra of the light output of several lamps.

Background
Study the theory of gratings and how the angle of diffracted light depends on the wavelength. How do the number of rulings per millimeter of the grating and the overall size of the grating affect the separation of the wavelengths?

Study the mechanical design of the spectrometer. The instrument consists of two mirrors and a grating, to direct and collimate light incident on an entrance slit, a photomultiplier (PMT) tube, placed at an exit slit, and a motor to scan the angle of the grating to send light of different wavelengths to the PMT. Look at the manual for the unit and find the page that displays the optical layout of the device. Gently take the cover off the spectrometer and look inside to identify all the key parts.

Experiments
Review the instructions for setting up and operating the spectrometer using a LabVIEW interface program running on the adjacent computer. These instructional sheets are located with the computer.
1) Getting Started
Using a mercury lamp, verify the calibration of the spectrometer by looking for the green line near 5450 Å. Use the CRC to get an accurate value for this emission wavelength in air. Take spectra with different slit widths and integration times. How do these parameters affect the intensity and wavelength resolution of the spectra you measure? How precise are the wavelengths you measure for the emission doublet of Sodium? How accurate are they? Discuss the distinction between accuracy and precision in this context.

2) Obtaining Hydrogen and Deuterium Spectra
Our goal is to obtain a measurement of the mass difference between hydrogen and deuterium by measuring the wavelength splitting between hydrogen and deuterium emission lines.

The energy levels of a hydrogen atom are given by

\[ E_n = -\frac{1}{2} Z^2 \alpha^2 \mu c^2 \frac{1}{n^2} \]  

(1)

where \( Z \) is the nuclear charge (\( Z=1 \) for H and D atoms), \( \alpha \) is the fine structure constant, \( \mu \) is the reduced mass \( \mu = m_e m_N / (m_e + m_N) \) (\( m_N \) is the mass of the relevant nucleus), \( c \) is the speed of light in vacuum, and \( n \) is the principal quantum number in the Bohr theory. The most recent adjusted values of the fundamental constants can be found on a sheet in this binder. The energies of corresponding levels in hydrogen and deuterium differ by approximately 0.25% because of the different reduced masses of these two atomic systems. Precise measurements of Balmer emission spectra for these atoms permit a determination of the deuteron-proton mass ratio \( m_d/m_p \) to an uncertainty of 1-2%.

In this experiment assume that Eq. (1) predicts the spectrum of hydrogen exactly and you will use your experimental spectra to determine the mass of the deuteron through its effect on the deuteron spectrum.

The energy levels involved in the Balmer \( \alpha, \beta, \gamma \) and \( \delta \) transitions of atomic hydrogen are shown in Fig 1. The vacuum wavelengths \( \lambda_{H \text{ vac}} \) of the Balmer \( H_\alpha, H_\beta, H_\gamma, \) and \( H_\delta \) lines for hydrogen, as calculated from Eq. (1) are given in Table I.
To obtain these wavelengths to six significant figures, the full precision of the fundamental constants is necessary. You will find it essential to maintain the precision of all numerical calculations to at least six significant figures to obtain meaningful results. Verify these values with your own calculations for at least two cases.

Eq. (1) determines transition wavelengths in vacuum, so a correction must be applied to obtain wavelengths as measured by a diffraction spectrometer in air at standard pressure and temperature. This correction is given by

$$\lambda_{\text{air}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{air}}}$$

where $n_{\text{air}}$ is the index of refraction of air. A useful tabulation of the index of refraction of air for visible light can be found in the CRC accompanying the experiment. Throughout your analysis treat this index of refraction correction as exact and do not estimate its negligible contribution to the uncertainty in your final experimental result for $m_{\text{air}}/m_p$. Again, verify the entries in the table for at least two cases.

Table I. Wavelengths of the first four Balmer transitions in vacuum and air at STP.

<table>
<thead>
<tr>
<th>Transition Name</th>
<th>$\lambda_{H \text{ vacuum}}$ (Å)</th>
<th>$\lambda_{H \text{ air}}$ (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H. $n=3 \rightarrow n=2$</td>
<td>6564.70</td>
<td>6562.89</td>
</tr>
<tr>
<td>H. $n=4 \rightarrow n=2$</td>
<td>4862.74</td>
<td>4861.38</td>
</tr>
<tr>
<td>H. $n=5 \rightarrow n=2$</td>
<td>4341.73</td>
<td>4340.51</td>
</tr>
<tr>
<td>H. $n=6 \rightarrow n=2$</td>
<td>4102.94</td>
<td>4101.78</td>
</tr>
</tbody>
</table>

\[\begin{array}{cccc}
n=6 & \text{-----} & \text{-----} & \text{-----} \\
n=5 & \text{-----} & \text{-----} & \text{-----} \\
n=4 & \text{-----} & \text{-----} & \text{-----} \\
n=3 & \text{-----} & \text{-----} & \text{-----} \\
n=2 & \text{-----} & \text{-----} & \text{-----} \\
n=1 & \text{-----} & \text{-----} & \text{-----} \\
\end{array}\]
Figure 1. Schematic Energy Level diagram for atomic hydrogen in the Bohr theory, with relevant Balmer transitions indicated. Energies are not to scale.
To obtain high-resolution spectra of the Balmer $\alpha, \beta, \gamma$ and $\delta$ lines of atomic and hydrogen and deuterium using the spectrometer controlled by the LabView program running on the computer arrange the spectrometer and lamps as shown in Figure 2.

![Diagram of spectrometer setup](image)

Figure 2. Top View of the H and D lamps on the optical table. The beamsplitter is used to combine the light output of the H and D lamps so that both their spectra can be acquired simultaneously.

Record all experimental parameters used including slit widths and integration times, in your notebook. The emission peaks should be Gaussian in shape. Plot spectra for all four sets of transitions by using Kaleidagraph. Avoid saturated peaks by reducing integration times and slits widths.

In the lamp discharges, the far fewer atoms are produced in the high n states so the Balmer $\delta$ lines will be much less intense than the Balmer $\alpha$ lines. It will be necessary to make adjustments in source position, slit widths and integration times to get well-resolved spectra for all four lines.

**Analysis**

1) From your measured spectra make a table that includes the peak position and its associated uncertainty. Use fits of Gaussian functions to your peaks to minimize your uncertainties in the peak centroid and its width. You should be able to locate the centroids of all peaks to within 0.05 Å. In the table record the full width at half maximum (FWHM) of each peak. Record also the slit widths used on the entrance and exit slits and the integration time used.
How do your measurement results compare with the calculated values in Table I? Is your agreement satisfactory given the manufacturer’s claim of a calibration accuracy of ± 0.2 Å for the spectrometer?

Use your measured values to calculate the wavelength separation, \( \Delta \lambda = \lambda_{\text{H}} - \lambda_{\text{D}} \), between the H and D peaks of the four Balmer spectra. Also calculate the associated uncertainties \( \delta(\Delta \lambda) \).

1) In your notebook, show that the deuteron-to-proton mass ratio can be determined from single Balmer transition using

\[
\frac{m_d}{m_p} = \frac{A_{\text{air}}}{A_{\text{air}} - \Delta \lambda_{\text{air}}} \tag{3}
\]

where \( A_{\text{air}} = \lambda_{\text{H}} - \lambda_{\text{H} \infty} \) is the theoretical wavelength separation (for air) between the hydrogen peak and the peak corresponding to the hypothetical, infinitely heavy nucleus case, \( \Delta \lambda_{\text{air}} = \lambda_{\text{H}} - \lambda_{\text{D}} \) is the experimental H-D peak separation from your measurements. Show also that the uncertainty in a particular \( m_d/m_p \) determination is given by

\[
\delta(\frac{m_d}{m_p}) = \frac{A_{\text{air}} \delta(\Delta \lambda_{\text{air}})}{(A_{\text{air}} - \Delta \lambda_{\text{air}})^2} \tag{4}
\]

Have a go at this derivation. Ask for help if you get stuck.

3) Calculate four values for the mass ratios from your four spectra. Place these results in a table. Compute a weighted mean of your mass ratio results and its associated uncertainty. Finally, compare this final result of the mass ratio with the accepted value.

Additional Experiments
Get a high precision value for the Rydberg constant from measurements of the spectrum of Hydrogen. Extend these measurements over as wide a range as possible given the sensitivity of the monochrometor’s detector.
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