Mentor: Paul Melvin

My research this summer will be on *3-dimensional manifolds*, spaces locally homeomorphic to 3-dimensional Euclidean space, and in particular on the computation of their *quantum invariants.* These remarkable invariants were introduced by Ed Witten in 1988 in the context of *quantum field theory*, defined rigorously by Reshetikhin-Turaev in 1989 using *quantum groups*, and extended and refined by Kirby-Melvin in 1991. Although still not very well understood, they are known to record very subtle characteristics of 3-manifolds.

The focus of my project will be on the computation of quantum invariants of an important class of 3-manifolds called *lens spaces*. The lens space L(p,q), where p and q are relatively prime integers, is obtained by gluing together two solid tori so that the meridian of one is identified with a curve on the other that wraps p times longitudinally and q times meridianally. The known formulas for quantum invariants of lens spaces involve basic number theoretic and topological notions (*Dedekind sums* and *mu-invariants*) that draw on tools that I learned about in my linear algebra course last year.

The first part of my project will be devoted to the study of the topological classification of lens spaces, due to Reidemeister in the 1930's and Moise in the 1950's. As a first step, I will write a Mathematica program to generate a list of the distinct lens spaces L(p,q) for any given p.

I will then study other descriptions of lens spaces (needed for understanding known formulas for their quantum invariants) as *surgery on the 3-sphere along a knot* (i.e., cutting out a tubular neighborhood of the knot and reattaching it in a different way) or *link* (a collection of knots), and will write a program to carry out the computation of their quantum invariants. In addition, I will attempt to flesh out a recent paper of Kenta Okazaki (2011) that gives closed formulas for the "spin-refined" invariants of Kirby-Melvin for lens spaces and investigate generalizations of these formulas to surgery on knots.