

Math B101: Calculus I

Calculus is our way of describing change. It teaches us how to measure the change of a variable over time (such as location, velocity, temperature, or monetary value). It allows us to predict changes in a system (as in climatology, economics, or probability). It even helps us choose optimal solutions to problems (as in machine learning or numerical analysis).

In this course we will cover the fundamental concepts of the *derivative* of a function, interpreting it both geometrically as the slope of a tangent line and quantitatively as the instantaneous rate of change. While learning *what* derivatives are and *how* to calculate them, we will discuss their applications to everyday problems. The course concludes by setting the stage for Calculus II; we will discuss the *antiderivative* and the *integral* of a function.

The prerequisite for this course is an SAT or ACT score that indicates proficiency in high school algebra, geometry, trigonometry, and precalculus. The minimum scores associated with this level of proficiency are an SAT of 630 or an ACT of 26. A score of 18 or higher on the Calculus Readiness Placement is also recommended.



B101-L Calculus 1 LAB

This lab course will reinforce the concepts and skills that are needed to be successful in Calculus 1. This class is geared toward students who have never seen Calculus before and/or students who have not taken a math class for a year or more.

Students must be enrolled in MATH B101 Calculus I to enroll in this course.



MATHB102: Calculus 2

Why? Calculus provides the language and tools necessary to understand and predict motion and change. Engineers, scientists, and economists use it to create modern technology, explain the natural world, and manage financial markets. Calculus is one of the greatest intellectual achievements in human history—without it, we would still be living in the Dark Ages. Mastering calculus not only opens doors to a variety of interesting majors and exciting careers, but also feels great!

What? This is the second in a sequence of two courses that covers single-variable calculus. Topics include techniques of integration, applications of integration, infinite sequences and series, tests of convergence for series, and power series.

Who? Prerequisite: a merit grade in Math 101 (or an equivalent experience).

MATH B104: PROBABILITY AND STATISTICS

There are no prerequisites for Math 104.

In a five-card poker hand what are the chances of getting different combinations of cards, such as getting all red cards or getting exactly three hearts? In this course you will learn how to calculate such chances and other similar ones. You will be surprised to discover from your work that some seemingly unlikely outcomes are more likely than you might think, while other outcomes that might seem likely actually have very little chance of occurring.

If you have two related sets of data, such as the heights and weights of a large group of people, you will learn how you can accurately estimate the weight of one of these people from the person's height, or estimate the height from the weight, just by knowing a few summary statistics about the whole group, such as the average height and average weight of the all the people.

Before elections we always hear polls about how voters are going to vote. How are these polls done accurately, and with such high confidence, by the pollsters? They are done by taking random samples of voters. We will learn how a sample can by drawn from a population, such as the population of voters, so that the sample will accurately reflect the population from which it is taken. We will learn the non-intuitive result that the sample can be very small and still produce very accurate results, provided the sample is taken in the right way. Then we will learn about how estimates and predictions about the voters can be made from the sample and how we can measure the accuracy of those results so that we can have high confidence in them.

The effectiveness of a new drug in treating a health condition is to be studied. How can the effectiveness of the drug be evaluated? How can we help prevent possible biases or confounding factors from affecting this evaluation? We will learn how this can be done and how the effectiveness can be mathematically evaluated by using what are called Tests of Significance, such as the z-test or the t-test.

These and other related topics will be studied in this course at an introductory level using only mathematical techniques with which you are familiar from earlier mathematics courses. The course will be done in "hybrid" format, so that you will be able to attend the course remotely on ZOOM.



MATHB201: Multivariable Calculus, Fall 2023 Mathematics Department, Bryn Mawr College

Prerequisite: Calculus I and II



Change is omnipresent. Weather patterns, the cost of textbooks, the number of people using Instagram, and our position in the universe are a few examples of things that are changing. **Calculus is the study of how things change.** It provides a framework for modeling systems that undergo change and for making predictions. Calculus has been applied to study and solve numerous problems in many areas including technology, science, and business.

Most functions that influence our lives involve multiple inputs. For example, the temperature you experience depends on your location on the earth, the time of day, and the day of the year. In multivariable calculus, we will learn how to analyze functions with multiple inputs.

From your earlier calculus courses, you should be familiar with ways to represent functions of a single variable and how to take derivatives and integrals of these single variable functions. In this course, you will learn:

- Several ways to represent functions of multiple variables;
- How to calculate and interpret partial derivatives;
- How to evaluate multivariable integrals and their applications.

Throughout the course, there will be an emphasis on both theory and applications. We will make use of Mathematica throughout the course to help understand and visualize multivariable calculus.

The text for the course is by Hughes-Hallett (and many other authors): *Multivariable Calculus, 6th Edition* **or** *Calculus: Single and Multivariable, 6th Edition* (Chapters 12 – 20)

Math B206: Transition to Higher Mathematics



This course is designed to help you transition from working with the computational topics that are typical of 100-level math courses to engaging with the more theoretical material found at the 300 level. It introduces content relevant to the three major types of mathematics that define the Bryn Mawr math major—algebraic, analytic, and applied—and also explores the culture and profession of mathematics.

Throughout the course, you will have the opportunity to:

- Understand the logic, structure, and techniques of mathematical proof.
- Learn concepts that are fundamental to algebra and analysis.
- Gain experience in typesetting mathematics and computer programming.
- Find and present information about the nature of mathematics, mathematical culture, professional organizations, or other aspects of mathematics as a human endeavor.

Topics encountered in this process include: sets, logic, counting, proof techniques, relations, functions, limits, typesetting with LaTeX, the R programming language and RStudio integrated development environment, and programming basics (conditional expressions, defining functions, and loops)

MATH B210: DIFFERENTIAL EQUATIONS WITH APPLICATIONS



How do we study functions when we ONLY KNOW THEIR DERIVATIVES?

DID YOU KNOW? Differential equations are used to study drug delivery, disease epidemics, chaos, electrical circuits, spacecraft navigation, supply & demand, and much more!

COURSE TOPICS: This focuses on using a combination of numerical/computational, geometric, and analytic methods to study various types of differential equations (linear vs. nonlinear; first-order vs. higher-order) and systems of equations. Various applications—especially in physics, biology, and economics—will also be explored.

Co-requisite: Math B201 (multivariable calculus), B203 (linear algebra), or instructor permission. Contact ejgraham@brynmawr.edu for more information.

Math Modeling and Sustainability

Math B295: Topics in Mathematics Fall 2023

Professor Victor Donnay, Bryn Mawr College Tuesday and Thursday, 2:25 p.m. – 3:45 p.m.

Pre-requisites: Calculus 101 and 102 or permission of the instructor.

In this course, we will use mathematics to study issues of sustainability. Is it possible to meet all of our energy needs using renewable energy? How much energy does a typical person in the United States use per year – for transportation, food production, home heating and cooling? What is the carbon footprint associated with this energy use? How much energy can be produced by solar panels and wind turbines (per square meter of installation) and so how many square miles of land would need to be devoted to solar and wind farms? By how much can we reduce energy use via conservation or new technologies? Are these approaches cost effective? What do we mean by "cost effective"?

To examine these issues, we will create mathematical models and using the modeling cycle in which one:

- Studies a real world situation;
- Makes simplifying assumptions about the real world situation and translates the simplified situation into mathematics.
- Analyzes the mathematics and determines what outcomes the model predicts.
- Compares the predicted outcomes with the real outcomes.
- Decides if the model is accurate enough for our needs. If not, repeat the cycle but now making a more complicated model.

The text for the course will be *Alternative Energy without the Hot Air* by David MacKay (available free on-line at <u>http://www.withouthotair.com/)</u>. We will also use sustainability modules developed by Tom Pfaff (<u>http://sustainabilitymath.org/</u>) that incorporate real world data. There will be weekly homework assignments on these materials and a midterm exam. We will make use of computer software (Excel, Mathematica) in our work; no previous computer experience is assumed.

Praxis Component: During the semester, students will work in teams to analyze a real world sustainability issue of interest to a community partner. Early in the term, the community partners will give presentations about their projects and students will select a project to work on. The project teams will be responsible for meeting with the community partner and for carrying out the project with support from Professor Donnay. During the semester, the teams will give progress reports on their projects to the class. The project will culminate with a final presentation to the community partner and a written report outlining the teams findings and recommendations. There will be a strong focus on developing written and oral communications skills so that one can clearly and effectively present convincing quantitative arguments.

MATH B301: REAL ANALYSIS I

Prerequisite: MATH B201 (Multivariable Calculus)

Note: This is a WA (Writing Attentive) course for the math major.

Analysis is the branch of mathematics that deals with inequalities and limiting processes. This is a first course in real analysis, providing a rigorous development of single variable calculus, with a strong focus on proof writing. In this course we shall study the theory of calculus, material that you may have seen in Math B101 (Calculus I), Math B102 (Calculus II), and Math B201. In those courses, the focus was on how to do various types of problems. In this course, the focus will be on understanding the underlying ideas of calculus using formal mathematical reasoning. We will start from the completeness of the real line, the distinguishing characteristic of analysis among other mathematical disciplines, and then we will continue with sequences, limits, topological properties of the real line (e.g. open sets, closed sets, limit points, closures, complements), and continuity.

Goals of the course include:

- To learn to communicate mathematical reasoning in writing and verbally, both via informal arguments and via more formal proofs;
- To understand and be able to use the fundamental theorems of real analysis.



This is a yearlong course in **Abstract Algebra**. As the oldest of the three fundamental pillars of Pure Mathematics – see the circular diagram above – algebra is both remarkably beautiful and widely applicable. The clarity gained by studying the subject from an abstract perspective will soon become apparent and will help us understand the concept of **symmetry** in mathematics and in nature. For example, there are exactly 60 symmetries of the dodecahedron shown above, together forming a remarkable a **group** known as A₅ that will feature at several points during the course; two other symmetry groups that are fundamental to the study of physics, SU(2) and SO(3), are featured on the pink face of the dodecahedron. This course will study many other notions, including **rings**, **fields**, **vector spaces**, **modules** and their **homomorphisms** (structure preserving maps), with the ultimate goal of unravelling the theory of equations. Here's one critical diagram that arises in group theory known as the "universal property of quotient groups". Ask us about it!



Take this course! You won't regret it.

Math B310: Mathematics of Financial Derivatives

Prerequisite: Math B201 (Multivariable Calculus) or its equivalent

This course is an introduction to the mathematics and models used in the valuation of financial derivatives (financial instruments whose values are based on that of one or more underlying assets, such as stocks, commodities, and currencies).

Concepts to be covered in this course include:

- Discrete Time Stochastic Processes and Pricing Models
- Black-Scholes Partial Differential Equation
- Hedging
- Martingale Representation Theorem.





Fall 2023 Course Announcement Math B398-001 Senior Conference:

EPIDEMIOLOGICAL MODELING IN AN

"INFODEMIC" AGE

Monday/Wednesday 10:10-11:30 AM Instructor: Erica Graham

COURSE DESCRIPTION

In the early days of the COVID-19 pandemic, the World Health Organization released a situation report addressing concerns of a related global problem: "a massive 'infodemic'-an over-abundance of information—some accurate and some not—that makes it hard for people to find trustworthy sources and reliable guidance when they need it." Mathematical models are commonly used to describe the spread of infectious disease within a population. In today's age of information overload, the challenge of disease intervention is exacerbated by an urgent need to counteract widespread falsehoods that can threaten public health, even when it seems the worst is behind us. In this course, we will use a combination of mathematical modeling, computer simulation, and data analysis to examine the interaction between infectious disease and information dynamics, as well as the efficacy of various control strategies.

NOTE: This course is only open to seniors majoring in mathematics at Bryn Mawr. Contact **ejgraham@brynmawr.edu** for more information.

$\mathfrak{a}_1 \subseteq \mathfrak{a}_2 \subseteq \mathfrak{a}_3 \subseteq \cdots$

 $\implies \mathfrak{a}_m = \mathfrak{a}_{m+1} = \cdots \text{ as } m \to +\infty$

What does it mean to know someone or something?

A pretty loaded question, right? It's both an academic and practical question with substantial theories developed across disciplines (seen to the right). In this course, we will grapple with this question and its possible answers from a mathematician's perspective. We will explore, through abstract algebra, the ways in which 20th century mathematicians thought about this question and how it informed their attempts to conjoin disparate areas of mathematics through common universal ideas.



MATH B503

Algebra I

Algebra itself, as a technical field, is "rooted" in problems of solving equations like

$$x^n+y^n=z^n$$
 and $ax^5+bx^4+cx^3+dx^2+ex+f=0$.

By the turn of the 20th century, we knew the latter was intimately related to **groups** and the former's study was most efficiently carried out with **rings**. Both are heavily related to **fields**. In historic terms, our course begins in a world where groups, rings, and fields are known already as important and interesting mathematical objects.



Our departure point for this **graduate course** is the systematic study of groups, rings, and fields in their own right, an undertaking which crested in the early 20th century. These advances occurred parallel to the invention of algebraic topology where groups were used to study flabby notions of shape and space. We will begin our studies with basic group theory and Hölder's program for classifying groups. Then, we will move to the commutative ring theory studied by Hilbert, Noëther, and many more. The semester will culminate with Galois theory, in near complete generality.

By the middle of the 20th century, it became clear that myriad advances in disparate fields were driven in part by **canonical** and **natural** associations between different **categories** of mathematical objects. The invention of **category theory** helped to unify mathematical branches through a common language of process and idea. So, a significant portion of our study will be focused on categorical thinking and **phenomena**.

Returning to our question, our answer will be borne out of category theory. And armed with the experience of asking and answering the question we will be ready to march on with a **21st century mathematical perspective!**

MATH B530: Differential Topology Mathematics Department, Bryn Mawr College

Prerequisite: Real Analysis II or permission of the instructor.



What comes to your mind when you think of Calculus? Perhaps you think of taking derivatives or doing integrals. It is a fascinating, extremely rich subject with countless applications. Much of calculus deals with smooth functions on standard Euclidean space. One generalization of this standard calculus is to generalize the types of functions being examined, a topic examined in graduate-level analysis. Another generalization is to generalize the domains of the functions, the focus of differential topology. In this course, we will learn how one can do *calculus on smooth manifolds*, which are spaces that locally like Euclidean space.

Differential topology is a basic foundation for many areas of current research in both the pure and applied realms. In this course, you will gain a deep understanding of the important concepts of Manifolds, Smooth Maps, and Integration on Manifolds. The primary text for the course is *Introduction to Smooth Manifolds, Second Edition* by John M. Lee.