If $\bar{x} = 5, s = 2$,

**a. What percentage of cases lie between 5 and 7?**

This z score corresponds to .1587 above the point of 7. And $0.5 - 0.1587 = 0.3413$, or 34.13% lie between 5 and 7.

**b. At what percentile are you at 7?**

$50.00 + 34.13 = 84.13$

**c. What percentage of cases lie above 9?**

$$z = \frac{9 - 5}{2} = \frac{4}{2} = 2,$$
which corresponds to .0228. So, 2.28% lie above 9.

**d. What percentage of cases lie below 9?**

$1 - 0.0228 = 0.9772$, or 97.72%.

**e. What percentage of scores lie between 3 and 7?**

$$z = \frac{7 - 5}{2} = \frac{2}{2} = 1,$$
which corresponds to .1587 in the table. This is the proportion of cases above 7. So the proportion of cases between 5 and 7 is $0.50 - 0.1587 = 0.3413$. Because the normal distribution is symmetrical, we only need to double this figure, $0.3413 + 0.3413$, to get the answer of $0.6826$, or 68.26% of cases.

**f. What percentage of cases lie between 1 and 7?**

We know that 34.13% lie between 5 and 7. Between 1 and 5 is

$$z = \frac{1 - 5}{2} = \frac{-4}{2} = -2,$$
so we are 2 SDs to the left of the mean. 2 SDs corresponds to a value of .0228, or $0.50 - 0.0228 = 0.4772$, or 47.72% of cases. So, add 34.13% + 47.72% and we get 81.85%.

**g. What percentage of cases lie between 3.2 and 4.5?**

$$z = \frac{4.5 - 5}{2} = \frac{-0.5}{2} = -.25,$$
which corresponds to a value of .4013. In other words, .0987 lie above this value of 4.5.

$$z = \frac{3.2 - 5}{2} = \frac{-1.8}{2} = -0.9,$$
which corresponds to .1841.

Because 9.87 lie above the maximum value and 18.41 percent lie below 3.2, or 28.28% lie either above or below these two values. Because the distribution is symmetrical, half of the distribution lies below the mean. Or, $50.00 - 28.28 = 21.72%$ of the distribution lies between these two values.
Confidence Interval Examples  
Vartanian, SW 540

1. $\bar{X}=10, s=3, n=400$

What is the 95% confidence interval for this mean?

A. Figure out the estimate of the standard error for the mean:

$$\sigma_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{3}{\sqrt{400}} = \frac{3}{20} = .15$$

B. The z value for the 95% confidence interval is 1.96, which has a value of .025 in the z value table, indicating that 2.5% of the distribution lies above this value at the top of the distribution and 2.5% of the distribution lies below this value at the bottom of the distribution.

$$\bar{X} \pm 1.96 \cdot \sigma_{\bar{X}}$$

or

$$10 + 1.96 \cdot .15 = 10 + .294 = 10.294$$

$$10 - 1.96 \cdot .15 = 10 - .294 = 9.706$$

We are 95% confident that the population mean lies between 9.706 and 10.294.

What is the 99% confidence interval for this mean?

We need to divide the 100%-99%=1% by 2 to find the appropriate z value in the table. ½ %= .5%, or .005.  We can look in the z table and find that the z value that corresponds to this is 2.58 (or 2.57).

We would use the same formula as above but replace the 1.96 with 2.58.

$$\bar{X} \pm 2.58 \cdot \sigma_{\bar{X}}$$

$$10 + 2.58 \cdot .15 = 10 + .387 = 10.387$$

$$10 - 2.58 \cdot .15 = 10 - .387 = 9.613$$

We are 99% confident that the population mean lies between 9.613 and 10.387.

The more confident we are, the less precise we are.

What is the 90% confidence interval for this mean?

We need to divide the 100%-90%=10% by 2 to find the appropriate z value in the table. 5 %= .05.  We can look in the z table and find that the z value that corresponds to this is 1.64 (or 1.65). So,

$$\bar{X} \pm 1.64 \cdot \sigma_{\bar{X}}$$

$$10 + 1.64 \cdot .15 = 10 + .246 = 10.246$$

$$10 - 1.64 \cdot .15 = 10 - .246 = 9.754.$$
Confidence Intervals for a Proportion  
Vartanian, SW 540

What is the 95% confidence interval for the following problem:

The estimated standard error for a proportion is

$$\sigma_k = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $\pi$ is the proportion of cases in the condition.

If the estimated proportion of people using some form or anti-depressant medication from a sample of 400 people is 40%, then $\pi = .40$, $(1-\pi) = .60$. So the standard error for the proportion is

$$\sigma_k = \sqrt{\frac{.4(.6)}{400}} = .024$$

We then use the following formula to determine the 95% confidence interval

$$\hat{p} \pm 1.96 \times \sigma_k$$

$$.4 + 1.96 \times .024 = .4 + .047 = .447$$

and

$$.4 - 1.96 \times .024 = .4 - .047 = .353.$$  

Determine the 99% confidence interval for this proportion.

$$\hat{p} \pm 2.58 \times \sigma_k$$

$$.4 + 2.58 \times .024 = .4 + .062 = .462$$

and

$$.4 - 2.58 \times .024 = .4 - .062 = .338.$$