Vartanian: Data 540

Confidence Intervals for b coefficients:

To get the confidence interval for b, we must determine the SE for b:

\[
\hat{\sigma}_b = \frac{\hat{\sigma}_{y/x}}{\sqrt{\frac{N}{\sum_{i=1}^{N} (X_i - \bar{X})^2}}}
\]

-- which means that we have to compute SE of Y/X.

\[
\hat{\sigma}_{y/x} = \sqrt{\frac{\sum_{i=1}^{N} (Y_i - \bar{Y})^2 - b \sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})}{N - 2}}
\]

Example:

<table>
<thead>
<tr>
<th>Experience</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>5</td>
<td>10,000</td>
</tr>
<tr>
<td>10</td>
<td>15,000</td>
</tr>
<tr>
<td>15</td>
<td>25,000</td>
</tr>
<tr>
<td>20</td>
<td>40,000</td>
</tr>
<tr>
<td>25</td>
<td>60,000</td>
</tr>
</tbody>
</table>

\[
N = 5 \ \bar{X} = 15 \ \bar{Y} = 30,000
\]
From doing a bunch of calculations, I’ll tell you that b = 2,500.

\[
\sum (y_i - \bar{y})^2 = (10,000 - 30,000)^2 + (15,000 - 30,000)^2 + (25,000 - 30,000)^2 + (40,000 - 30,000)^2 + (60,000 - 30,000)^2 = 1,650,000,000
\]

\[
\sum (x_i - \bar{x})(y_i - \bar{y}) = (5 - 15)(10,000 - 30,000) + (10 - 15)(15,000 - 30,000) + (15 - 15)(25,000 - 30,000) + (20 - 15)(40,000 - 30,000) + (25 - 15)(60,000 - 30,000) = 200,000 + 75,000 + 0 + 50,000 + 300,000 = 625,000
\]

Therefore, the standard error for b =

\[
\hat{\sigma}_b = \frac{1,650,000,000 - 2,500(625,000)}{3} = \sqrt{29,066,666.67} = 5,400
\]

To get the 95% confidence interval for b with N-2 DFs, or 3 DFs, we use the t table in the back of the book.

2,500 ± (3.182) (341.77)

[3.182 is the critical value for 95% confidence interval.]

2,500 ± 1087.5 =

1412.5 to 3587.5
Thus, we are 95% confident that the real value of B is between 1412 and 3587.50.

Because 0 is not in this interval, we would reject the null hypothesis that B = 0.

**Direct way to Test if B is significant**

$H_0: B = 0.$

$H_A: B > 0.$

or

$H_A: B \neq 0.$

$$t_{n-2} = \frac{b_1}{SE_{b_1}}$$

$$t_3 = \frac{2.500}{341.27} = 7.33$$

For a one-tailed, 5% test, the critical value with 3 DFs is 2.353. Because the t value is greater than the critical value, we will reject the null hypothesis.

For a two-tailed test, the critical value with 3 DFs is 3.182. Because the t value is greater than the critical value, we will reject the null hypothesis.