3.2 Sliding blocks with friction

Mass $M_A = 4 \text{ kg}$ rests on top of mass $M_B = 5 \text{ kg}$ that rests on a frictionless table. The coefficient of friction between the two blocks is such that the blocks just start to slip when the horizontal force $F$ applied to the lower block is $27 \text{ N}$. Suppose that now a horizontal force is applied to the upper block. What is its maximum value for the blocks to slide without slipping relative to each other?

\[ f = m_A a \Rightarrow a = \frac{f}{m_A} \]

\[ F - f = m_B a \]

eliminate $a$

\[ F - f = \frac{m_B f}{m_A} \]

\[ F = f \left(1 + \frac{m_B}{m_A}\right) = f \left(\frac{m_B + m_A}{m_A}\right) \]

\[ f = \frac{M_A F}{M_A + M_B} \]

Now apply force $F'$ to top block instead

\[ F' - f = m_A a' \]

\[ f = M_B a' \Rightarrow a' = \frac{f}{M_B} \]

eliminate $a'$

\[ F' - f = \frac{M_A f}{M_B} \]

\[ F' = f \left(1 + \frac{M_A}{M_B}\right) = f \left(\frac{M_A + M_B}{M_B}\right) \]

\[ f = \frac{F' M_B}{M_A + M_B} \]

Set two equal since

\[ \frac{M_A F}{M_A + M_B} = \frac{M_B F'}{M_A + M_B} \]

\[ \Rightarrow F' = \frac{M_A F}{M_B} = \frac{4}{5} 27 \text{ N} = 21.6 \text{ N} \]
- 3.4 Synchronous orbit

Find the radius of the orbit of a synchronous satellite that circles the Earth. (A synchronous satellite goes around the Earth every 24 h, so that its position appears stationary with respect to a ground station.) The simplest way to find the answer and give your results is by expressing all distances in terms of the Earth's radius $R_e$.

\[ r_s = \frac{2\pi R_e}{T} \]  

\[ F = ma \]

\[
\frac{G M e m}{r_s^2} = m \frac{v^2}{r_s} = \frac{r_e^2 (G - m) m}{r_e^2 r_s^2} = m \frac{4 \pi^2 r_s^2}{r_s T^2} = \frac{4 \pi^2 r_s^2}{r_e T^2}
\]

\[
g = 9.8 \frac{m}{s^2} \]

\[
T = 24 \text{ hr} \times \frac{3600 \text{ s}}{1 \text{ hr}}
\]

\[
R_e = 6.4 \times 10^6 \text{ m}
\]

\[
r_s = R_e \left( \frac{G T^2}{4 \pi^2 r_e} \right)^{1/3}
\]

\[
r_s = 6.5 \times 10^6 \text{ m} = 42 \times 10^6 \text{ km}
\]
Mass and axle

A mass $m$ is connected to a vertical revolving axle by two strings of length $l$, each making an angle of $45^\circ$ with the axle, as shown. Both the axle and mass are revolving with angular velocity $\omega$. Gravity is directed downward.

(a) Draw a clear force diagram for $m$.
(b) Find the tension in the upper string, $T_{up}$, and lower string, $T_{low}$.

\[
\begin{align*}
T_1 + T_2 &= m l \omega^2 \\
T_1 / \sqrt{2} + T_2 / \sqrt{2} &= m r \omega^2 \\
T_1 / \sqrt{2} - T_2 / \sqrt{2} &= -m g
\end{align*}
\]

\[
\begin{align*}
T_1 &= \frac{1}{2} m (l \omega^2 + \sqrt{2} g) \\
T_2 &= \frac{1}{2} m (l \omega^2 - \sqrt{2} g)
\end{align*}
\]
A block rests on a wedge inclined at angle $\theta$. The coefficient of friction between the block and plane is $\mu$.

(a) Find the maximum value of $\theta$ for the block to remain motionless on the wedge when the wedge is fixed in position.

(b) The wedge is given horizontal acceleration $a$, as shown. Assuming that $\tan \theta > \mu$, find the minimum acceleration for the block to remain on the wedge without sliding.

(c) Repeat part (b), but find the maximum value of the acceleration.
3.17 Turning car

A car enters a turn whose radius is $R$. The road is banked at angle $\theta$, and the coefficient of friction between the wheels and the road is $\mu$. Find the maximum and minimum speeds for the car to stay on the road without skidding sideways.

\[ a = \frac{v^2}{R} \]

\[ \Sigma F_x = ma \]
\[ N \sin \theta - f \cos \theta = \frac{mv^2}{R} \]

\[ N \sin \theta - \mu N \cos \theta = \frac{mv^2}{R} \]  

\[ \Sigma F_y = 0 \]
\[ N \cos \theta + f \sin \theta - mg = 0 \]
\[ N \cos \theta + \mu N \sin \theta = mg \]

divide $\text{I}/\text{II}$

\[ \frac{N \sin \theta - \mu N \cos \theta}{N \cos \theta + \mu N \sin \theta} = \frac{mv^2}{R \cdot mg} \]

\[ \frac{v_{\text{min}}}{v_{\text{max}}} = \sqrt{gR \left( \frac{\tan \theta - \mu}{1 + \mu \tan \theta} \right)} \]
\[ v_{\text{max}} = \sqrt{gR \left( \frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right)} \]
Find the frequency of oscillation of mass $m$ suspended by two springs having constants $k_1$ and $k_2$, in each of the configurations shown.

(a) When mass is moved a dist $x$, spring 2 stretches by dist $x-d$ and spring 1 by dist $d$.

So on the mass spring 2 pulls us force

$$F_2 = -k_2 (x-d)$$

at point of connection between springs.

Spring 2 pulls us force $F_2 = k_2 (x-d)$

and spring 1 w/ force $F_1 = -k_1 d$.

Set the sum of these two to zero (assuming massless springs)

$$k_2 x - k_2 d - k_1 d = 0$$

Then solve for $d$

$$k_2 x = (k_2 + k_1) d \implies d = \frac{k_2 x}{k_1 + k_2}$$

Put this into $F_2$

$$F_2 = -k_2 \left( x - \frac{k_2}{k_1 + k_2} \right) = -k_2 \frac{k_2}{k_1 + k_2} \left( \frac{k_1 + k_2}{k_2} \right) x$$

$$F_2 = -k_{eff} x$$

So

$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 k_2}{(k_1 + k_2) m}}$$
b) \[ b_2 x - b_1 x = -(b_1 + b_2) x \]
\[ = -k_{eff} x \]

so \[ w = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 + k_2}{m}} \]