Abstract:
Some definitions of the word symmetry include “correct or pleasing proportion of the parts of a thing,” “balanced proportions,” and “the property of remaining invariant under certain changes, as of orientation in space.” One might think of snowflakes, butterflies, and our own faces as naturally symmetric objects – or at least close to it. Mathematically one can also conjure up many symmetric objects: even and odd functions, fractals, certain matrices, and modular forms, a type of symmetric complex function. All of these things exhibit a kind of beauty in their symmetries, so would they lose some of their innate beauty if their symmetries were altered? Alternatively, could some measure of beauty be gained with slight symmetric imperfections? We will explore these questions guided by the topic of modular forms and their variants. What can be gained by perturbing modular symmetries in particular? We will discuss this theme from past to present: the origins of these questions have their roots in the first half of the 20th century, dating back to Ramanujan and Gauss, while some fascinating and surprising answers come from just the last 15 years.