Abstract:
In order to understand and distinguish complicated topological spaces, we often compute algebraic invariants: if two spaces have different invariants, then they are certainly different themselves. (For example, for those who recognize them: the Euler characteristic of a surface, the Alexander polynomial of a knot, the fundamental group of a manifold.) But one might also wonder about the converse: are there algebraic invariants that completely determine something about the topological structure of the space? We will talk about this question in dimension four, where the answer is a resounding, "Sometimes!" Knots, surfaces, and 4-dimensional space will all play important roles.