The Empty Place in My Space: How \( \mathbb{Q} \) Complete Me

Daniel White

We all know one metric on the rationals: the one induced by absolute value. Did you know there are others? For each prime \( p \) there is an associated metric on \( \mathbb{Q} \), induced by what’s known as the p-adic norm. These appear naturally when one attempts to classify the norms that can be placed on \( \mathbb{Q} \) and are the only others that exist up to some equivalence. The rabbit hole doesn’t end there! Remember that \( \mathbb{R} \) is the completion of \( \mathbb{Q} \), i.e. what appears when you “fill in the holes”. There’s nothing stopping us from completing \( \mathbb{Q} \) when equipped with these new fancy metrics, and we obtain fields known, each known as the p-adic numbers.

At this talk, we will discover how the p-adic norms appear as we classify all norms on \( \mathbb{Q} \), a result due to Ostrowski. We’ll then discuss one method of constructing \( \mathbb{R} \) from \( \mathbb{Q} \) with the absolute value and use that same construction to produce the p-adic numbers.

Date: Wednesday February 13, 2019
Time: 7:00 pm
Place: Park 328