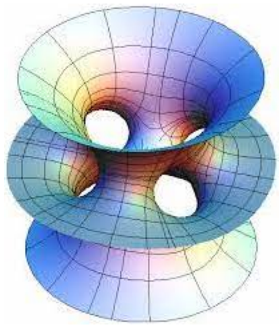
Math B322/522: Complex Analysis

Bryn Mawr College

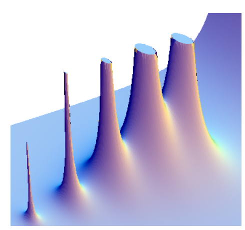


Complex analysis is the calculus of *complex functions*, in which all variables may take on values that are complex numbers rather than just real numbers. In this course, we focus on complex functions of a *single variable* (as in your first calculus course) and engage in a study of their derivatives and integrals. As complex numbers correspond to the points in the *plane* rather than a *line*, complex functions have graphs that live in four rather than two dimensions. Thus alternative methods for visualizing such functions must be developed.

One remarkable difference between real and complex analysis is that all complex *differentiable* functions are in fact *analytic*, thus infinitely differentiable and locally expressible as power series. This is in stark contrast with real functions. Consequently these functions are better behaved than most real functions. For example, they are uniquely determined by their values on arbitrarily small regions in their domains! This adds a marvelous geometric flavor to the subject.

Here is a rough outline for the course:

- Analytic Functions
  a) Complex Numbers
  b) Complex Functions
  c) Continuity
  d) Differentiability
- Integration
  a) Contour Integrals
  b) Cauchy's Theorem
  c) Cauchy Integral Formula
  d) Applications
- Series
  a) Basic Concepts
  b) Taylor's and Laurent's Theorems
  c) Power Series and Laurent Series
  d) Zeros and Isolated Singularities
- Calculus of Residues
  a) Computing Residues
  b) Residue Theorem
  c) Real Integrals and Series
- Conformal Mappings
  a) Riemann Mapping Theorem
  b) Fractional Linear Transformations
  c) Applications



Broader Impact of the Course Complex analysis is widely applicable both within mathematics (in number theory, algebraic and differential geometry, combinatorics, and applied mathematics) and outside mathematics, especially in physics (hydrodynamics, thermodynamics, and quantum mechanics) and nuclear, aerospace, mechanical and electrical engineering.

Prerequisites Required: Multivariable Calculus Recommended: Real Analysis

The 322/522 Divide All students will attend the same lectures. Those enrolled in Math B322 will have extra problem sessions to work on homework. Those in Math B522 (which carries graduate credit) will have extra discussion sessions to cover more theoretical aspects of the course (such as the proof of the Riemann Mapping Theorem) and to broaden the scope of the course (including topics such as Riemann surfaces, the Weierstrass Factorization Theorem, and the Picard Theorems).