Abstract:

The symmetric functions (SYM) are the set of polynomials with variables in \( \{x_1, \ldots, x_n\} \) which are unchanged by exchanging any variable \( x_i \) with \( x_j \). As a vector space, the symmetric functions have a number of nice “famous” bases, including: the monomials, the complete homogeneous functions, the elementary bases, the Schur functions and the power sums. These nice, combinatorially defined bases encode many of the tricks in a combinatorics class, and give a basic way to explore representation theory of the symmetric group. We'll describe these bases, and continuing work to explore their analogs in two related spaces: the quasisymmetric functions (QSYM) and noncommutative symmetric functions (NSYM). Until recently, the power sums did not have a well-studied analogue in QSYM, so we will specifically highlight joint work with Ballantine, Daugherty, Mason, and Niese which explores the properties of such an analogue. We'll assume some basic linear algebra, but we otherwise aim for a self-contained talk.