MATH B303: ABSTRACT ALGEBRA I

Prerequisite: Math B203, Linear Algebra

MATH B304: ABSTRACT ALGEBRA II

Prerequisite: Math B303, Abstract Algebra

If you enjoy linear algebra, then you will probably also enjoy abstract algebra. Many of the algebraic concepts to which you are introduced in the particular context of linear algebra are studied in greater generality in abstract algebra. For example, in linear algebra you study vector spaces, which are sets having two binary operations, addition and scalar multiplication, that are required to satisfy certain properties. In abstract algebra you also will study sets on which are defined one or more binary operations that are required to satisfy certain properties. The names of some of these types of sets that you will study are **groups**, **rings**, **fields**, and **integral domains**, among others. In linear algebra you study the functions between vector spaces, called linear transformations, which have special important properties related to the addition and scalar multiplication of the vector spaces. Similarly in abstract algebra we will study functions between groups or between rings, etc. that have special properties related to the binary operations of our sets, and we generally call these functions **homomorphisms**. In linear algebra we study subspaces of vector spaces, which are smaller vector spaces within the larger one. We learn that many sets of special interest, such as kernels and images of linear transformations of vector spaces, are always subspaces. In abstract algebra we do the same thing and study **subgroups** of groups or **subrings** of rings, etc. in the same way and learn again that many sets of special interest, such as important ones related to homomorphisms, are always subgroups, subrings, etc.

In Math 303 we will study many particular groups, rings, fields and integral domains of interest, such as groups related to permutations or to symmetries of regular polygons or rings that involve modular arithmetic. Later in Math 304 we will also study polynomials and the nature of their roots, by applying much of what we have learned in Math 303.

Many interesting questions can be investigated using abstract algebra. Here are a few examples that can be explained easily here.

- Many groups (mentioned above) contain only a finite number of elements. If this number is prime, such as 7 or 29, then we can state exactly what properties this group must have and we can also show that there is essentially only one group having that particular number of elements.

- We know that the roots of a quadratic polynomial can be computed using the quadratic formula, and this formula involves only basic arithmetic and square roots. Could we find similar formulas to compute roots of polynomials of degrees higher than two? In some cases yes, but we can use abstract algebra to show that there cannot be such a formula that works in general for polynomials of degree 5!

- In high school geometry you learn about constructions using straightedge and compass, such as constructing the perpendicular bisector of a line segment. But some seemingly simple constructions turn out to be impossible in general, and abstract algebra can be used to prove this. For example, using abstract algebra it can be proved that a 60° angle cannot be trisected using only a straightedge and compass.

These are just a few of the interesting problems that abstract algebra allows us to investigate.