"Peaks and descents of permutations"

Monday, September 12, 2016

Talk at 4:00 – H109
Tea at 3:30 – KINSC Math Lounge, H208

Abstract:
Given a permutation $\pi$ in $S_n$, we say an index $i$ is a peak if $\pi(i-1) < \pi(i) > \pi(i+1)$. Let $P(\pi)$ denote the set of peaks of $\pi$. Given any set $S$ of positive integers, Billey, Burdzy, and Sagan showed that the number of permutations in $S_n$ with peak set $S$ is given by a polynomial (depending on $S$) times a power of two. They conjectured that the coefficients of this polynomial expanded in a binomial coefficient basis centered at $\max(S)$ are all nonnegative. In this talk we prove that their "positivity conjecture" is true. It remains an open question to find a combinatorial meaning of these non-negative coefficients. Near the end of the talk, we will discuss various current developments regarding this topic, including some similar questions replacing "peaks" by "descents." No prerequisites.