Abstract: It is a classical theorem in the theory of modular forms that the points \( \frac{x}{\sqrt{N}} \), where \( x \in \mathbb{Z}^n \) runs over all the solutions to \( \sum_{i=1}^n x_i^2 = N \), equidistribute on \( S^{n-1} \) for \( n \geq 4 \) as \( N \) (odd) tends to infinity. The rate of equidistribution poses however a more challenging problem. Due to its Diophantine nature the points inherit a repulsion property, which opposes equidistribution on small sets. Sarnak conjectures that this Diophantine repulsion is the only obstruction to the rate of equidistribution. Using the smooth delta-symbol circle method, developed by Heath-Brown, Sardari was able to show that the conjecture is true for \( n \geq 5 \) and recovering Sarnak’s progress towards the conjecture for \( n = 4 \). Building on Sardari’s work, Browning, Kumaraswamy, and myself were able to reduce the conjecture to correlation sums of Kloosterman sums of the following type:

\[
\sum_{q \leq Q} \frac{1}{q} S(m, n; q) \exp(4\pi i \alpha \sqrt{mn}/q).
\]

Assuming the twisted Linnik conjecture, which states that the above sum is \( O((Qmn)^e) \) for \( |\alpha| \leq 2 \), we are able to verify Sarnak’s Conjecture. I shall lose a few words on the unconditional progress towards this conjecture and how (unfortunately) it is insufficient to improve unconditionally what is known towards Sarnak’s conjecture. If time permits, I will talk about ongoing research of how the automorphic approach and the circle method approach may be combined to hopefully give better insight into Sarnak’s conjecture.