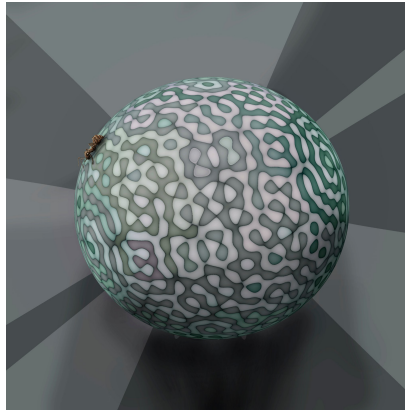


# Math 501: Graduate Real Analysis I

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Quick, take a guess:

- Can you take the usual ball (as in, the basketball from the gym), split it into finitely many pieces, move them around without stretching, and assemble the pieces back into two balls of the same size as the original one?
- Can you write an infinite two-dimensional array of numbers so that, when summed over rows first, they add up to 5, but summed over columns first, they add up to 12?
- Will I be able to do mathematics without worrying about all of this stuff?<sup>1</sup>

We will be concerned with the central notions of *measure* and *integration*. This theory forms the language and the backbone of modern mathematical analysis.

It is perhaps possible to study some questions in analysis relying only on the Riemann integral, but this is true in the same sense in which it is possible to study geometry without drawing pictures or travel the world while being only able to speak in Latin. Measure theory and functional analysis enable a “complete” theory of Fourier series and integrals, with its vast applications to spectral analysis and wave and image processing, underpins entire probability theory (where measure is simply called “probability”), the study of long-term evolution and chaos known as ergodic theory, the entire modern mathematical economics, and so on.

Hardy famously wrote, “Beauty is the first test. There is no permanent place in the world for ugly mathematics”. The theory of measure and integration passed this test with flying colors a century ago and moved on to swiftly change the face of analysis. I hope to get you excited about the different viewpoint that it provides.

**Pre-requisites:** Students should be comfortable with core undergraduate Analysis, including limits, continuity, differentiability, Riemann integration, and series. At Bryn Mawr, these prerequisites will be amply fulfilled by the sequence of Math 301 and 302. A student who has completed Math 301 and has had other substantial exposure to proof-based mathematics should also be able to transition into Graduate Analysis without trouble (talk to the professor).

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<sup>1</sup> Correct answers: Yes, Yes, and Yes (after this course)!