Abstract Algebra II
Math B304

This is the second semester of our yearlong course in Abstract Algebra. In it we reencounter many of the now familiar objects from the first semester course – groups, rings and fields and their homomorphisms, as well as some basic notions from linear algebra – vector spaces and linear transformations – and their generalizations – modules and their homomorphisms. It will be seen how all of these structures interrelate, and how they bear on some classical problems in algebra and geometry.

For example, we will explore generalizations of the integers – including the Gaussian and Eisenstein integers (pictured on the right above), prove a general structure theorem for finitely generated modules over principal ideal domains (after explaining what those terms mean!), which will be seen to imply the classification of finite abelian groups (treated in the first semester) as well as the existence of certain canonical forms for matrices (the Jordan and rational forms). We will see how the structure of fields and their “extensions” translates into the impossibility of solving the ancient problem of trisecting an angle with a straightedge and compass. And we will explore a deep relationship between the structure of groups and their subgroups, and the structure of fields and their extensions (Galois theory), and see how this implies the impossibility of finding a formula for the roots of a polynomial of degree five or higher.

Come and enjoy the ride!

\[ \cdots \rightarrow V_{k-1} \rightarrow V_k \rightarrow V_{k+1} \rightarrow \cdots \]