Philadelphia Area Number Theory Seminar

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Ribet's Lemma, the Brumer-Stark Conjecture, and the Main Conjecture

Abstract: In 1976, Ken Ribet used modular techniques to prove an important relationship between class groups of cyclotomic fields and special values of the zeta function. Ribet's method was generalized to prove the Iwasawa Main Conjecture for odd primes p by Mazur-Wiles over $\mathbb{Q}$ and by Wiles over arbitrary totally real fields.

Central to Ribet's technique is the construction of a nontrivial extension of one Galois character by another, given a Galois representation satisfying certain properties. Throughout the literature, when working integrally at p, one finds the assumption that the two characters are not congruent mod p. For instance, in Wiles' proof of the Main Conjecture, it is assumed that p is odd precisely because the relevant characters might be congruent modulo 2, though they are necessarily distinct modulo any odd prime.

In this talk I will present a proof of Ribet's Lemma in the case that the characters are residually indistinguishable. As arithmetic applications, one obtains a proof of the Iwasawa Main Conjecture for totally real fields at $p = 2$. Moreover, we complete the proof of the Brumer-Stark conjecture by handling the localization at $p = 2$, building on joint work with Mahesh Kakde for odd p. Our results yield the full Equivariant Tamagawa Number conjecture for the minus part of the Tate motive associated to a CM abelian extension of a totally real field, which has many important corollaries.

This is joint work with Mahesh Kakde, Jesse Silliman, and Jiuya Wang.

Wednesday, March 15, 2023
2:00–4:00 PM

Temple University
Tuttleman Hall, Room **404**

Informal refreshments at 2:00PM – Talk at 2:30PM