Abstract:

A guiding question in number theory, specifically in the subfield called arithmetic statistics, is: How many number fields are there? Number fields are vector spaces over the rational numbers that include both the rational numbers and the roots of a fixed polynomial $f(x)$ under the operations of addition and multiplication. Like other fields, every nonzero element of a number field has a multiplicative inverse. If we allow ourselves to vary over polynomials of a fixed degree $n$, we can refine the question to the way number theorists like to study it: How many number fields of a fixed dimension $n$ are there? And, if we filter the family of polynomials not only by degree but on the types of symmetries the roots have dictated by what's known as the Galois group, we arrive at the questions surrounding Malle's Conjecture: precisely, how does the count of number fields of degree $n$ whose normal closure has Galois group $G$ grow as their discriminants tend to infinity? In this talk, we will discuss the history of this question including its connection to the inverse Galois problem and take a closer look at the story in the case that $n = 2,3,4$, i.e. the counts of quadratic, cubic, and quartic fields.