

# *A Morsel from Euler*

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Consider the infinite series

$$\begin{aligned} \tan(\pi/4) + \frac{1}{2}\tan(\pi/8) + \frac{1}{4}\tan(\pi/16) \\ + \frac{1}{8}\tan(\pi/32) + \frac{1}{16}\tan(\pi/64) + \dots \end{aligned}$$

Here are three questions: Can you show that this series converges? Can you show it converges to something less than 2? Can you find the EXACT value of the series?

Students who have mastered the comparison test should be able to answer the first two. But the third – determining the sum exactly – seems next to impossible. Who could possibly do such a thing?

The answer is Leonhard Euler, the foremost mathematician of the 18<sup>th</sup> century. Using logarithms, derivatives, a few trig identities, and something he called “infinite numbers,” Euler succeeded in finding the sum. This was a spectacular example of his mathematical agility.

If you want to see how he did it, come to the **first DMC of the year** and match wits with one of history’s greatest thinkers.

**Date: September 12**

**Time: 7:00 pm**

**Place: Park 338**

Pre-requisites: Calculus I and II