A Tale of Two Series

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Infinite series received their first serious treatment when Jakob Bernoulli published his *Tractatus de seriebus infinitis* in 1689. Ever since, this subject has been a staple of calculus courses, here at Bryn Mawr and beyond.

But if you think you know all there is about infinite series, you might be in for a surprise. Leonhard Euler, the greatest mathematician of the 18th century, found ways to sum the seemingly unsummable. In this regard, I have a pair of examples in which his genius is on full display.

The first is the “Basel problem,” where Euler determined the *exact* value of

\[
1 + 1/4 + 1/9 + 1/16 + 1/25 + \ldots + 1/n^2 + \ldots
\]

I'll present not his original derivation from 1734 but a lesser-known argument from 1755 in which he (improbably) used l'Hospital’s rule.

Building on this, Euler evaluated a second infinite series so bizarre that finding its exact sum seemed hopeless. But he did it. As my second example, this stands as a jaw-dropping demonstration of mathematical talent.

If you want to look over the shoulder of one of history's greatest mathematicians, please stop by the Distressing Math Collective (DMC) on Thursday October 12 at 7 p.m. in Park 338.

NOTE: The talk will be accessible to anyone who has completed the calculus sequence.