Counting Local Systems on Supersingular Abelian Varieties

Abstract: Given a supersingular abelian variety $A$ defined over an algebraically closed field of characteristic $p$, we count the number of homomorphisms from the étale fundamental group of $A$ to $GL_n(q)$, where $q$ is a power of $p$. This count (for fixed $n$) turns out to be a polynomial in $q$. We will give a somewhat complicated formula for this polynomial, then state a few theorems which elucidate its features, including a new result about semidirect products of profinite groups.