Abstract

We study the complete Kaluza-Klein expansion of Yang-Mills theory on a compact manifold. The moduli space of Yang-Mills gauge potentials is a principal fiber bundle, whose metric determines the kinetic terms for charged scalar fields in the Kaluza-Klein expansion. We present an expression for the physical metric on moduli space in terms of fiber metric, compensator field (geometrically, the bundle connection) and base metric, taking motivations from the case of U(1) gauge theory. Here, we will be identifying those compensators in case of U(1) gauge theory, and D-dimensional Yang-Mills theory, it is necessary to introduce “compensator fields”. Now, consider the full space

\[ \mathfrak{L}_m = \mathfrak{g}_m \mathfrak{g}_n (\mathfrak{g}_m \mathfrak{g}_n)^{-1} \]

where \( \mathfrak{g}_m \) and \( \mathfrak{g}_n \) are the horizontal and vertical moduli, and the scalers deforming away from flat \( F_{mn}=0 \) are massive.

KK Expansion in Yang-Mills Theory

We will follow similar steps as in U(1) case. So we proceed keeping our considerations same as before.

Letting \( \mathfrak{a}(\mathfrak{g}) \) denote the coordinates on \( \mathfrak{A}(\mathfrak{g}) = \mathfrak{g}^{-1}(v)\mathfrak{A}(\mathfrak{g}(v)) \mathfrak{g}(v) \), and \( \mathcal{A}_m(\mathfrak{g}) \) denotes a fiducial representative of each gauge field, we have the horizontal tangent bundle

\[ \mathfrak{A}(\mathfrak{g}) = \{ \mathfrak{a}(\mathfrak{g}) \} \]

The moduli space of Yang-Mills gauge potentials is a principal fiber bundle, whose connection \( \mathfrak{g} = \mathfrak{g}_{mn} \mathfrak{g}_m \mathfrak{g}_n \) is the physical space of equivalence classes of gauge connections and the physical metric

\[ \mathcal{G}(\mathfrak{g}) = \mathcal{G}(\mathfrak{g})_{mn} \mathfrak{g}_m \mathfrak{g}_n \]

\[ \mathcal{G}(\mathfrak{g})_{mn} = \mathfrak{g}^{-1}(v)\mathfrak{g}_m \mathfrak{g}_n (\mathfrak{g}^{-1}(v)\mathfrak{g}_n \mathfrak{g}_m)^{-1} \]

\[ \mathfrak{g}^{-1}(v)\mathfrak{g}_m \mathfrak{g}_n (\mathfrak{g}^{-1}(v)\mathfrak{g}_n \mathfrak{g}_m)^{-1} = \mathfrak{g}^{-1}(v)\mathfrak{g}_m \mathfrak{g}_n (\mathfrak{g}^{-1}(v)\mathfrak{g}_n \mathfrak{g}_m)^{-1} \]

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