

Philadelphia Area Number Theory Seminar

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Quadratic Identities and Maass Waveforms II

Abstract: Andrews, Dyson, and Hickerson (ADH) studied the Fourier coefficients of the function

$$\sigma(q) = 1 + \sum_{n=1}^{\infty} \frac{q^{n(n+1)/2}}{(1+q)(1+q^2)\cdots(1+q^n)},$$

where σ is a function that appears in the work of Ramanujan. They prove, among other things, that, for $m > 0$ and $m \equiv 1 \pmod{24}$, that these Fourier coefficients are given by

$$T(m) = \# \left\{ \begin{array}{l} \text{equivalence classes } [(x, y)] \text{ of solutions to} \\ x^2 - 6y^2 = m \text{ with } x + 3y \equiv \pm 1 \pmod{12} \end{array} \right\} \\ - \# \left\{ \begin{array}{l} \text{equivalence classes } [(x, y)] \text{ of solutions to} \\ x^2 - 6y^2 = m \text{ with } x + 3y \equiv \pm 5 \pmod{12} \end{array} \right\}.$$

Cohen showed that

$$\phi_0(\tau) = y^{1/2} \sum_{\substack{n \in \mathbb{Z} \\ n \neq 0}} T(n) e^{2\pi i n x / 24} K_0 \left(\frac{2\pi |n| y}{24} \right)$$

is a Maass waveform on $\Gamma_0(2)$. Zweger was able to place $\phi_0(\tau)$ in a larger framework of indefinite theta functions.

In this talk, I will discuss the problem of placing quadratic identities arising in the work of ADH into a modular framework. This is joint work, in progress, with Larry Rolin.

Wednesday, February 21, 2018, 2:40 – 4:00 PM

Bryn Mawr College, Department of Mathematics

Park Science Center 328 · Tea and refreshments at 2:20PM in Park 339