Philadelphia Area Number Theory Seminar

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Intrinsic Diophantine Approximation on S^3

Abstract: It is a classical theorem in the theory of modular forms that the points $\boldsymbol{x}/\sqrt{N}$, where $\boldsymbol{x} \in \mathbb{Z}^n$ runs over all the solutions to $\sum_{i=1}^{n} x_i^2 = N$, equidistribute on S^{n-1} for $n \geqslant 4$ as N (odd) tends to infinity. The rate of equidistribution poses however a more challenging problem. Due to its Diophantine nature the points inherit a repulsion property, which opposes equidistribution on small sets. Sarnak conjectures that this Diophantine repulsion is the only obstruction to the rate of equidistribution. Using the smooth delta-symbol circle method, developed by Heath-Brown, Sardari was able to show that the conjecture is true for $n \geqslant 5$ and recovering Sarnak's progress towards the conjecture for $n = 4$. Building on Sardari's work, Browning, Kumaraswamy, and myself were able to reduce the conjecture to correlation sums of Kloosterman sums of the following type:

$$\sum_{q \leq Q} \frac{1}{q} S(m, n; q) \exp(4\pi i \alpha \sqrt{mn}/q).$$

Assuming the twisted Linnik conjecture, which states that the above sum is $\mathrm{O}((Qmn)^{\epsilon})$ for $|\alpha| \leqslant 2$, we are able to verify Sarnak's Conjecture. I shall lose a few words on the unconditional progress towards this conjecture and how (unfortunately) it is insufficient to improve unconditionally what is known towards Sarnak's conjecture. If time permits, I will talk about ongoing research of how the automorphic approach and the circle method approach may be combined to hopefully give better insight into Sarnak's conjecture.

Wednesday, October 24, 2018, 2:40 – 4:00 PM

Bryn Mawr College, Department of Mathematics
Park Science Center **328** · Tea and refreshments at 2:20PM in Park 361